BC Calc Vector Practice Name: Block: Seat:

1. The velocity of a particle moving in the xy-plane is given by $\langle 3t^2 + 2t - 6, 2\cos(\pi t)\rangle$
   (a) Find the speed at time $t = 3$

$$\|\vec{v}(3)\| = \| \langle 3(3)^2 + 2(3) - 6, \ 2\cos(3\pi)\rangle \|$$

$$= \sqrt{27^2 + (-2)^2} = \sqrt{733}$$

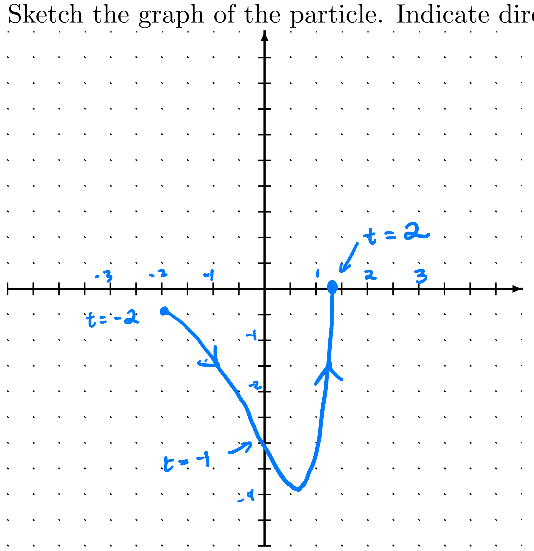
   (b) Find the position vector $\mathbf{r}(t)$ if the particle is at $(5, -6)$ when $t = 1$. $(\vec{s}(1) = \langle 5, -6\rangle)$

$$\vec{s}(t) = \langle 5 + \int_1^t 3t^2 + 2t - 6 \, dt, \ -6 + \int_1^t 2\cos \pi t \, dt\rangle$$

$$\vec{s}(t) = \langle 5 + (t^3 - t^2 - 6t) - (3+2-1), \ -6 + (\tfrac{2}{\pi}\sin \pi t - (-2))\rangle$$

$$\vec{s}(t) = \langle t^3 - t^2 - 6t + 1, \ \tfrac{2}{\pi}\sin(\pi t) - 4\rangle$$

2. The position vector of a particle is given by $\mathbf{r}(t) = \langle \ln(t + 2), t^2 - 4\rangle$ for $-2 < t \leq 2$.
   (a) Sketch the graph of the particle. Indicate direction.



   (b) Find the acceleration vector at any time t.

$$\vec{v}(t) = \langle \frac{1}{t+2}, \ 2t\rangle$$

$$\vec{a}(t) = \langle \frac{d}{dt}(t+2)^{-1}, \ 2\rangle$$

$$\vec{a}(t) = \langle -(t+2)^{-2}, \ 2\rangle$$

$$\vec{a}(t) = \langle \frac{-1}{(t+2)^2}, \ 2\rangle$$

typo if

3. Given $\frac{d\mathbf{r}}{dt} = 4e^{2t}\mathbf{i} + 6t\mathbf{j}$ Find $\mathbf{r}(t)$ ~~is~~ $\mathbf{r}(0) = 5\mathbf{i} - 8\mathbf{j}$.

Method 1: $\vec{r}(t) = \int \frac{d\vec{r}}{dt}\, dt$

$$\vec{r}(t) = \langle 2e^{2t} + C_1, 3t^2 + C_2 \rangle$$

$2e^0 + C_1 = 5$ so $C_1 = 3$

$3(0)^2 + C_2 = -8$ so $C_2 = -8$

$$\vec{r}(t) = \langle 2e^{2t} + 3, 3t^2 - 8 \rangle$$

Method 2:

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(t)\, dt$$

$$= \langle 5, -8 \rangle + \left\langle 2e^{2t}\Big|_0^t, 3t^2\Big|_0^t \right\rangle$$

$$= \langle 5, -8 \rangle + \langle 2e^{2t} - 2, 3t^2 \rangle$$

$$= \langle 2e^{2t} + 3, 3t^2 - 8 \rangle$$

A moving particle has the position $(5, -6)$ at $t = 1$, and the velocity vector at any given time $t > 0$ is given by $\left\langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right\rangle = \vec{v}(t)$

4. (a) Find the acceleration vector at time $t = 3$.

by calc: $\left\langle \frac{d}{dt}\left(1 - \frac{1}{t^2}\right)\Big|_{t=3}, \frac{d}{dt}\left(2 + \frac{1}{t^2}\right)\Big|_{t=3} \right\rangle$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \left\langle \frac{2}{t^3}, -\frac{2}{t^3} \right\rangle$$

$$\vec{a}(3) = \left\langle \frac{2}{27}, -\frac{2}{27} \right\rangle \text{ or } \frac{2}{27}\left(\vec{i} - \vec{j}\right)$$

(b) Find the position vector at time $t = 3$.

Method 1:

$$\vec{s}(t) = \langle 5, -6 \rangle + \int_1^t \left\langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right\rangle dt$$

$$s(t) = \langle 5, -6 \rangle + \left\langle \left(t + \frac{1}{t}\right) - (1+1), \left(2t - \frac{1}{t}\right) - (2-1) \right\rangle$$

$$\vec{s}(t) = \left\langle t + \frac{1}{t} + 3, 2t - \frac{1}{t} - 7 \right\rangle$$

$$\vec{s}(3) = \left\langle 3 + \frac{1}{3} + 3, 6 - \frac{1}{3} - 7 \right\rangle$$

Method 2:

$$\vec{s} = \left\langle t + \frac{1}{t} + C_1^{=3}, 2 - \frac{1}{t} + C_2 \right\rangle$$

$$1 + \frac{1}{1} + C_1 = 5, \quad C_1 = 3$$

$$2 - 1 + C_2 = -6, \quad C_2 = -7$$

$$\vec{s} = \left\langle t + \frac{1}{t} + 3, 2t - \frac{1}{t} - 7 \right\rangle$$

$$\vec{s}(3) = \left\langle \frac{19}{3}, -\frac{4}{3} \right\rangle$$

(c) For what time t, for $t > 0$, does the line tangent to the path of the particle at $\mathbf{r}(t)$ have a slope of 8?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = \frac{2t^2 + 1}{t^2 - 1} = 8$$

at time $t \approx \underline{1.224}7449$

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

$$\lim_{t \to \infty} \frac{2t^2 + 1}{t^2 - 1} = \lim_{t \to \infty} \frac{2 + 1/t^2}{1 - 1/t^2} = 2$$

 page 841 (11e) Thm 12.3 Position vector for a projectile

5. A projectile is launched at a height of 16 feet above the ground with an initial velocity of 70 feet per second at an angle of 30 degrees above the horizontal.

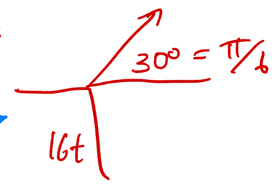
h v_0 $\theta = \pi/6$

(a) Find the vector valued function for the path.

$$\vec{r}(t) = \left\langle (v_0 \cos\theta)t,\ h + (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \right\rangle$$

$$\vec{r}(t) = \left\langle \left(70\cos\tfrac{\pi}{6}\right)t,\ 16 + \left(70\sin\tfrac{\pi}{6}\right)t - \tfrac{1}{2}(32)t^2 \right\rangle$$

$$\vec{r}(t) = \left\langle 35\sqrt{3}\,t,\ 16 + 35t - 16t^2 \right\rangle$$



(b) At what time does it reach the maximum height?

$$\vec{v}(t) = \left\langle 35\sqrt{3},\ 35 - 32t \right\rangle$$

$$y'(t) = 35 - 32t$$

$35 - 32t = 0$

$t = \frac{35}{32}$

y': $+$ $\frac{35}{32}$ $-$

The max is $\frac{35}{32}$ sec. since the

(c) What is the maximum height?

$$16 + 35\left(\tfrac{35}{32}\right) - 16\left(\tfrac{35}{32}\right)^2 \text{ feet}$$

$$\left(= \frac{2249}{32} = 35.140625 \text{ feet}\right)$$

(d) How long will it take to reach the ground (height=0)?

$$y = 16 + 35(t) - 16t^2 = 0$$

at $t = 2.57573$ second.

(e) How far (horizonatally) will it be from where it was launched?

$$x = (35\sqrt{3})(2.575738213)$$

about 156.1458308 feet